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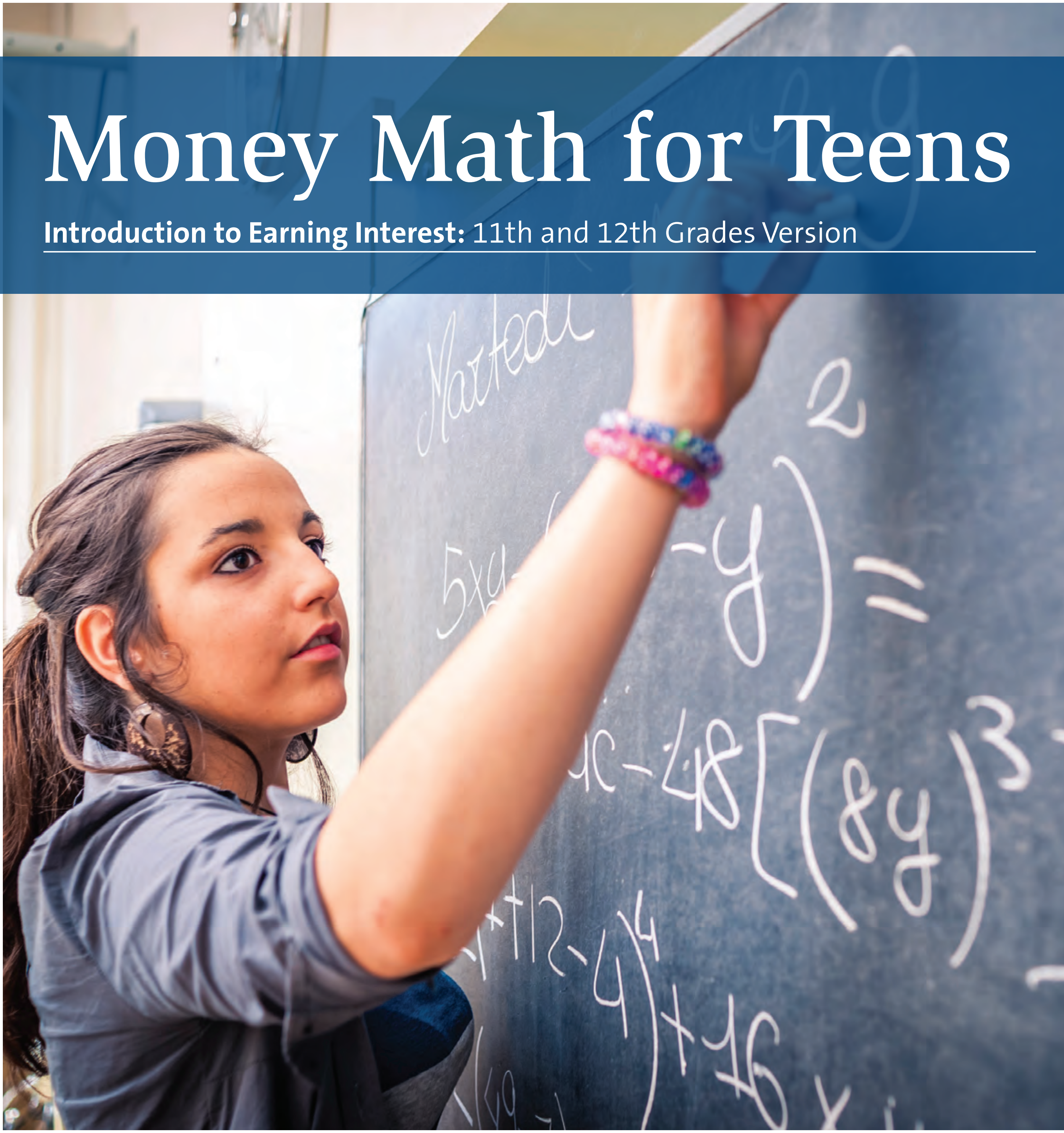
Money Math for Teens

Introduction to Earning Interest: 11th and 12th Grades Version



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Introduction to Earning Interest: APR, APY and Compound Interest

11th and 12th Grades Lesson Plan

OBJECTIVE

Saving and investing money safely and wisely are critical skills for people of all ages and backgrounds. Armed with the knowledge of how investments work, savvy investors can make informed decisions and determine the best investment choices available to them.

Students will:

- Know and be able to use investment vocabulary
- Know and be able to use the formula for calculating compound interest
- Understand the effect of compounding on savings.

TEACHING MATERIALS

- Lesson plan
- Teacher worksheet with precalculated examples
- Now You Try student worksheet with solutions
- Student assessment worksheet with solutions

LESSON ACTIVITY

1. **Discuss vocabulary words principal, deposit, interest, term, APR and APY.**
2. **Certificate of deposit (CD):**
   - Compounds annually.
   - Work through precalculated examples of annual compounding (see teacher worksheet, Examples 1 and 2). Example 2 shows compounding over multiple years.
   - Now You Try student worksheet: practice multiyear annual interest calculations (page 10).
3. **Statement savings account:**
   - Compounds quarterly.
   - Work through precalculated example of quarterly compounding (see teacher worksheet, Example 3).
     - *Note:* The calculations presented do not take into account that different months have different numbers of days. Quarterly calculations are done by computing annual interest, then dividing by 4.
   - Emphasize that annual interest paid quarterly > annual interest paid annually.
   - Define and demonstrate how to calculate APY.
   - Now You Try student worksheet: practice quarterly interest calculations (page 10).

4. **Compound interest formula:**
   - Introduce the compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where: ***A*** **=** Accumulated balance
P **=** Principal
r **=** APR expressed as a decimal
n **=** Number of compounding periods per year
t **=** Number of years the investment lasts

   - Review the scenario outlined in Example 3, using the compound interest formula to calculate Michael's principal, interest and APY at the end of one year of quarterly compounding.
     - The formula produces the same accumulated value **($8,161.20).**
     - The interest is still **$161.20.**
     - The APY is still **2.015%.**
   - Now You Try student worksheet: practice annual vs. quarterly interest calculations using the compound interest formula (page 11).

5. **Money market account:**
   - Compounds monthly.
   - Work through precalculated example of monthly compounding (see teacher worksheet, Example 4).
     - *Note:* The calculations presented do not take into account that different months have different numbers of days. Monthly calculations are done by computing annual interest, then dividing by 12.
   - Recalculate using the compound interest formula.

6. **Annual vs. quarterly vs. monthly compounding:**
   - If the principal and APR of investments that compound annually, quarterly and monthly are the same, which investment will have the greatest return in interest (i.e., APY)?
     - Show that monthly compounding yields highest APY.
   - Now You Try student worksheet: practice annual, quarterly and monthly interest calculations using the compound interest formula (page 12).

7. **Discussion:**
   - Why might a bank advertise deposit accounts using APY instead of APR?
   - Why might they advertise APR instead of APY on credit cards?

8. **Evaluate students' comprehension (see assessment worksheet).**

Teacher Worksheet

Vocabulary

Principal: An amount of money owned by an investor and held by a financial institution such as a bank.

Deposit(s): The act of establishing, or adding to, existing principal in an account (verb); the money placed in the account (noun).

Interest: The amount of money you earn by leaving deposits in a bank or financial institution. Interest is a percentage of your principal.

Term: The period of time an investment lasts.

Annual percentage rate (APR): The percentage rate at which interest is calculated annually.

Certificate of deposit (CD): An agreement between an investor and a bank (or financial institution) whereby the investor agrees to put a certain amount of money on deposit (this is the principal) for a certain amount of time without withdrawing it (this is the term) and the bank agrees to pay the investor interest at an agreed-upon percentage rate, known as the **annual percentage rate (APR).**

Compounding period: The amount of time that elapses between interest payments.

- Annual compounding: once per year
- Quarterly compounding: once every three months
  - January – March: 1st quarter
  - April – June: 2nd quarter
  - July – September: 3rd quarter
  - October – December: 4th quarter
- Monthly compounding: once per month

Compound interest: Interest calculated on both the principal you have on deposit and on interest that has accumulated in the past.

Principal and Interest

Example 1

Michael is saving money to buy a car. He takes **$8,000** to the bank and opens an annual CD upon which the bank agrees to pay him **2%** interest.

Principal = 8000
Term = 1 year
APR = 2% = 0.02
8000 x 0.02 = **$160**
8000 + 160 = **$8,160**

Beginning Balance	2% Interest	Ending Balance
$8,000	$160	$8,160

After one year, Michael has earned **$160** in interest on his initial deposit of $8,000, so his balance is now **$8,160**.

Annual Compounding: Certificate of Deposit

Example 2

Now, let's say Michael leaves his money in the bank for four years. The term of the annual CD is four years, so he will be earning **2%** interest per year for **four years.** Since this is an annual CD, interest will be added to the principal at the end of every year. This is called **annual compounding.**

	Beginning Balance	2% Interest	Ending Balance
Year 1	$8,000.00	$160.00	$8,160.00
Year 2	$8,160.00	$163.20	$8,323.20
Year 3	$8,323.20	$166.46	$8,489.66
Year 4	$8,489.66	$169.79	$8,659.45

Quarterly Compounding: Statement Savings Account

Example 3

Michael's bank offers other types of investment accounts in addition to certificates of deposit. One such type of account is a **statement savings account.** This type of account is similar to a certificate of deposit in that it also pays an annual percentage rate (APR) of interest, but there are some differences, too.

A statement savings account doesn't require Michael to promise not to take the money out for a specific period of time. Michael can go in and withdraw his money any time he wants. Also, instead of paying Michael his interest only once per year (annually), the bank will make an interest payment deposit into his account at the end of every quarter, because statement savings accounts compound quarterly.

When the bank makes an interest payment, the interest Michael earned during that quarter is added to his principal, and the new balance becomes Michael's new principal balance for the next quarter. Now Michael will begin earning interest on his interest! This is called **compound interest.**

Let's say Michael takes his $8,000 to the bank and opens a statement savings account instead of a certificate of deposit. The bank is going to pay him the same 2% interest on this account that it was offering for the CD.

Michael's beginning principal amount is again **$8,000.** Also, his APR is still **2%.** However, Michael gets **interest compounded quarterly** on this account.

At the end of the 4th quarter, what will Michael's principal balance be?

1. First, calculate his annual interest: 8000 x 0.02 = $160
2. Next, calculate what his 1st quarter interest payment will be: 160/4 = $40
3. **At the end of the 1st quarter,** Michael's new principal balance will be **$8,040.**
4. Next, calculate the annual interest he will earn on $8,040: 8040 x 0.02 = $160.80
5. Calculate his 2nd quarter interest payment: 160.80/4 = $40.20
6. **At the end of the 2nd quarter,** Michael's new principal balance will be 8040 + 40.20 = **$8,080.20.**
7. Next, calculate the annual interest he will earn on $8,080.20: 8080.20 x 0.02 = $161.60
8. Calculate his 3rd quarter interest payment: 161.60/4 = $40.40
9. **At the end of the 3rd quarter,** Michael's new principal balance will be 8080.20 + 40.40 = **$8,120.60.**
10. Finally, calculate the annual interest he will earn on $8,120.60: 8120.60 x 0.02 = $162.41
11. Calculate his 4th quarter interest payment: 162.41/4 = $40.60
12. **At the end of the 4th quarter,** Michael's new principal balance will be 8120.60 + 40.60 = **$8,161.20.**

After four quarters have passed, Michael has had his money in the statement savings account for one year. His ending balance at the end of that year is **$8,161.20.**

Do you remember what his ending balance would have been if he had opened an annual CD instead? **($8,160)**

Let's compare these two choices.

- Which type of account would have earned Michael more interest? **(Statement savings account)**
- How much *more* interest would Michael earn by opening the statement savings account instead of the annual CD? **($1.20)**

Why? After all, *both* accounts pay the same 2% APR.

If 2% of $8,000 is $160, and he earned $161.20 on his principal in the statement savings account, then he must have actually earned *more* than 2% in the statement savings account.

This is because he earned interest on his interest during the year. His statement savings account yielded more than 2% for the year. This extra earning because of compounding interest is called **annual percentage yield,** or **APY.** APY is the actual rate your money earns, taking compounding into consideration.

To calculate the APY, we divide the amount of interest Michael earned for the year by his original principal deposit:

$$\text{APY} = \frac{161.2}{8000} = 0.02015 \mathbf{= 2.015\%}$$

So a statement savings account that pays an **APR of 2%** will earn an **APY of 2.015%** because of the effect of compound interest.

Now You Try

Ask students to do page 10 of the Now You Try student worksheet to practice calculating multiyear annual compound interest and quarterly compound interest.

Compound Interest Formula

As you can see, compounding several times per year and holding an investment for multiple years would make for a lot of manual calculations. Luckily, there is a formula called the **compound interest formulA** that allows us to calculate the accumulated balance of an investment across multiple years and multiple compounding periods.

Introduce the compound interest formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Where: ***A* =** Accumulated balance

***P* =** Principal

***r* =** APR expressed as a decimal

***n* =** Number of compounding periods per year

***t* =** Number of years the investment lasts

Review the scenario outlined in Example 3:

Let's say Michael takes his $8,000 to the bank and opens a statement savings account instead of a certificate of deposit. The bank is going to pay him the same 2% interest on this account that it was offering for the CD.

Michael's beginning principal amount is again **$8,000.** Also, his APR is still 2%. However, Michael gets **interest compounded quarterly** on this account.

Principal (P) = 8000

APR (r) = 2% = 0.02

n = 4 (quarterly)

t = 1 year

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = 8000\left(1 + \frac{0.02}{4}\right)^{4\text{x}1}$$

$$A = 8000(1 + 0.005)^4$$

$$A = 8000(1.005)^4$$

$$A = 8000(1.02015)$$

A = **$8.161.20**

Compare the interest calculated using the formula with the interest calculated when we did the calculation the long way, by hand. Notice that the amount of interest (**$161.20**) matches.

Now You Try

Ask students to do page 11 of the Now You Try student worksheet.

Monthly Compounding: Money Market Savings Account

Example 4

Michael's bank offers another type of investment account similar to the statement savings account. This account is called a **money market savings account.** This type of account works just like a statement savings account except that the compounding period is monthly instead of quarterly. This means that Michael will receive an interest payment deposit into his account at the end of every month.

When that happens, the interest Michael earned in the previous month is added to his principal, and the new balance becomes Michael's new principal balance for the next month. So now Michael will begin earning interest on his interest monthly!

This time, Michael takes his $8,000 to the bank and opens a money market savings account instead of a statement savings account. The bank is going to pay him the same 2% on this account that it was offering for the statement savings account.

Michael's beginning principal amount is again **$8,000.** Also, his APR is still **2%.** However, Michael gets **interest compounded monthly** on this account.

After three months, or one quarter, what will Michael's principal balance be?

1. First, calculate his annual interest: 8000 x 0.02 = $160
2. Next, calculate what his 1st month's interest payment will be: 160/12 = $13.33
3. **At the end of the 1st month,** Michael's new principal balance will be **$8,013.33.**

If we continue for all 12 months of the year:

	Beginning Balance	2% Interest	Ending Balance
1st month	$8,000.00	$13.33	$8,013.33
2nd month	$8,013.33	$13.36	$8,026.69
3rd month	$8,026.69	$13.38	$8,040.07
4th month	$8,040.07	$13.40	$8,053.47
5th month	$8,053.47	$13.42	$8,066.89
6th month	$8,066.89	$13.44	$8,080.33
7th month	$8,080.33	$13.47	$8,093.80
8th month	$8,093.80	$13.49	$8,107.29
9th month	$8,107.29	$13.51	$8,120.80
10th month	$8,120.80	$13.53	$8,134.33
11th month	$8,134.33	$13.56	$8,147.89
12th month	$8,147.89	$13.58	$8,161.47

Note: We rounded *up* the monthly interest calculation before we added a month's interest to the balance at the beginning of the month. There may be as much as a $0.03 difference at the end of the year, as can be seen below in the accumulated balance calculated by using the formula. This $0.03 difference still yields the same APY.

Michael's \$8,000 original principal deposit, put into a money market savings account at 2% APR, compounding monthly, would be worth **\$8,161.47** at the end of one year.

What APY does this account yield? Again, divide the amount of interest Michael earned for the year by his original principal deposit:

$$\text{APY} = \frac{161.44}{8000} = 0.02018 = \mathbf{2.018\%}$$

Now, let's try using the formula to calculate Michael's balance at the end of one year:

Principal (P) = 8000

APR (r) = 2% = 0.02

n = 4 (quarterly)

t = 1 year

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 8000\left(1+\frac{0.02}{12}\right)^{12\text{x}1}$$

$$A = 8000(1 + 0.001666)^{12}$$

$$A = 8000(1.001666)^{12}$$

$$A = 8000(1.02018)$$

$$A = \mathbf{\$8.161.44}$$

The formula yields an accumulated balance of **\$8,161.44**—or \$0.03 less than it would be if the calculation were done by hand.

Compare Annual vs. Quarterly vs. Monthly Compounding

We've seen that the length of the compounding period for an investment affects its APY. If the APR for an investment is 2%, the APY will be:

- 2% for annual compounding
- 2.015% for quarterly compounding
- 2.018% for monthly compounding.

Now You Try

Ask students to do page 12 of the Now You Try student worksheet.

Discussion Questions

- Why might a bank advertise deposit accounts using APY instead of APR?
  - Deposit account interest earns people money, so they would be attracted to the higher rate.
- Why might they advertise APR instead of APY on credit cards?
  - Credit card interest costs people money, so they would rather see the lower rate.

Assessment

Ask students to complete the assessment worksheet.

Name Date

Student Worksheet

NOW YOU TRY: APR, APY AND COMPOUND INTEREST

Annual and Quarterly Interest

Taylor wants to invest her **$7,500** college fund at her bank. She picks an annual certificate of deposit that will pay her **3%** interest each year. She won't need her college money for three more years.

(Remember, 3% is 0.03 when written as a decimal.)

A. Fill in the table to find out how Taylor's investment grows.

	Beginning Balance	3% Interest	Ending Balance
Year 1	**$7,500.00**		
Year 2			
Year 3			

B. If Taylor chose a statement savings account that compounds *quarterly* instead, how would her account grow in the first year?

	Beginning Balance	3% Interest	Ending Balance
1st quarter	**$7,500.00**		
2nd quarter			
3rd quarter			
4th quarter			

C. Compare the APYs Taylor's accounts would earn depending on which account type she chooses.

Annual CD APY:

Quarterly statement savings account APY:

Name Date

Annual vs. Quarterly Interest: Compound Interest Formula

There is a formula you can use to calculate the ending balance of an investment if you know certain facts about the investment, such as its principal, APR, compounding periods and the number of years the investment lasts.

$$A = P\left(1+ \frac{r}{n}\right)^{nt}$$

Where: A **=** Accumulated balance

P **=** Principal

r **=** APR expressed as a decimal

n **=** Number of compounding periods per year

t **=** Number of years the investment lasts

Let's look again at Taylor's situation:

Taylor wants to invest her **$7,500** college fund at her bank. She has two investment options: a CD that compounds annually, or a statement savings account that compounds quarterly. Both are offering an APR of **3%.** She won't need her college money for three more years.

(Remember, 3% is 0.03 when written as a decimal.)

2. What are the...

Principal =

APR =

Number of compounding periods per year =

Number of years the investment lasts = **3**

... for the CD and the statement savings account?

Use the compound interest formula to calculate her balance at the end of the three-year investment period for both the annual CD and the quarterly statement savings account.

Annual CD

$$A = P\left(1+ \frac{r}{n}\right)^{nt}$$

Quarterly Statement Savings Account

$$A = P\left(1+ \frac{r}{n}\right)^{nt}$$

Name Date

Annual vs. Quarterly vs. Monthly Interest: Compound Interest Formula

Andrea wants to invest **$2,500** at her bank. She has three investment options: a CD that compounds annually, a statement savings account that compounds quarterly or a money market savings account that compounds monthly. All are offering an APR of **3%.** She won't need the funds in her account for five more years.

(Remember, 3% is 0.03 when written as a decimal.)

A. Use the compound interest formula to calculate her balance at the end of the five-year investment period for each option: the annual CD, quarterly statement savings account and monthly money market savings account.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Annual CD **Quarterly Statement Savings Account** **Monthly Money Market Savings Account**

B. Explain what investment advice you'd give Andrea and why.

Student Worksheet: Solutions

NOW YOU TRY: SOLUTIONS

Annual and Quarterly Interest

1. Taylor wants to invest her **$7,500** college fund at her bank. She picks an annual certificate of deposit that will pay her **3%** interest each year. She won't need her college money for three more years.

(Remember, 3% is 0.03 when written as a decimal.)

A. Fill in the table to find out how Taylor's investment grows.

	Beginning Balance	3% Interest	Ending Balance
Year 1	**$7,500.00**	**$225.00**	**$7,725.00**
Year 2	**$7,725.00**	**$231.75**	**$7,956.75**
Year 3	**$7,956.75**	**$238.70**	**$8,195.45**

B. If Taylor chose a statement savings account that compounds *quarterly* instead, how would her account grow in the first year?

	Beginning Balance	3% Interest	Ending Balance
1st quarter	**$ 7,500.00**	**$56.25**	**$7,556.25**
2nd quarter	**$7,556.25**	**$56.67**	**$7,612.92**
3rd quarter	**$7,612.92**	**$57.10**	**$7,670.02**
4th quarter	**$7,670.02**	**$57.53**	**$7,727.55**

C. Compare the APYs Taylor's accounts would earn depending on which account type she chooses.

Annual CD APY:

$$\text{APY} = \frac{225}{7500} = 0.03 \;\mathbf{= 3\%}$$

Quarterly statement savings account APY:

$$\text{APY} = \frac{227.55}{7500} = 0.03034 \;\mathbf{= 3.034\%}$$

Annual vs. Quarterly Interest: Compound Interest Formula

There is a formula you can use to calculate the ending balance of an investment if you know certain facts about the investment, such as its principal, APR, compounding periods and the number of years the investment lasts.

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Where: ***A*** **=** Accumulated balance

P **=** Principal

r = APR expressed as a decimal

n = Number of compounding periods per year

t = Number of years the investment lasts

Let's look again at Taylor's situation:

Taylor wants to invest her **$7,500** college fund at her bank. She has two investment options: a CD that compounds annually, or a statement savings account that compounds quarterly. Both are offering an APR of **3%.** She won't need her college money for three more years.

(Remember, 3% is 0.03 when written as a decimal.)

2. What are the...

Principal = **7500**

APR = **3% = 0.03**

Number of compounding periods per year = **1 for the CD, 4 for the statement savings account**

Number of years the investment lasts = **3**

... for the CD and the statement savings account?

Use the compound interest formula to calculate her balance at the end of the three-year investment period for both the annual CD and the quarterly statement savings account.

Annual CD

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 7500\left(1+\frac{0.03}{1}\right)^{1\text{x}3}$$

$$A = 7500(1.03)^3$$

$$A = 7500(1.092727)$$

$A =$ **$8.195.45**

Quarterly Statement Savings Account

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = 7500\left(1+\frac{0.03}{4}\right)^{4\text{x}3}$$

$$A = 7500(1 + 0.0075)^{12}$$

$$A = 7500(1.0075)^{12}$$

$$A = 7500(1.093806)$$

$A =$ **$8.203.55**

Annual vs. Quarterly vs. Monthly Interest: Compound Interest Formula

3. Andrea wants to invest **$2,500** at her bank. She has three investment options: a CD that compounds annually, a statement savings account that compounds quarterly or a money market savings account that compounds monthly. All are offering an APR of **3%.** She won't need the funds in her account for five more years.

(Remember, 3% is 0.03 when written as a decimal.)

A. Use the compound interest formula to calculate her balance at the end of the five-year investment period for each option: the annual CD, quarterly statement savings account and monthly money market savings account.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Annual CD	**Quarterly Statement Savings Account**	**Monthly Money Market Savings Account**
$A = 2500\left(1 + \frac{0.03}{1}\right)^{1x5}$	$A = 2500\left(1 + \frac{0.03}{4}\right)^{4x5}$	$A = 2500\left(1 + \frac{0.03}{12}\right)^{12x5}$
$A = 2500(1.03)^{5}$	$A = 2500(1 + 0.0075)^{20}$	$A = 2500(1 + 0.0075)^{60}$
$A = 2500(1.159274)$	$A = 2500(1.0075)^{20}$	$A = 2500(1.0025)^{60}$
$A =$ **$2,898.19**	$A = 2500(1.161184)$	$A = 2500(1.161617)$
	$A =$ **$2,902.96**	$A =$ **$2,904.04**

B. Explain what investment advice you'd give Andrea and why.

Andrea should put her money into the money market savings account that compounds monthly because it will yield the greatest return.

Name Date

Student Assessment

Saving money and planning for your future are not only smart things to do, they're essential! Making smart decisions about investments begins by educating yourself about what types of investments are available to you and which will work the best for you in your particular situation.

The compound interest formula is a tool you can use to evaluate different investments. Make it work for you.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Goal: You want to have **$10,000** saved in a five-year time frame.

How much principal would you need to put away in a bank account offering a **4%** APR and compounding monthly in order to reach that goal?

A =

P =

r =

n =

t =

Challenge

You have **$8,250** to invest, and you won't need the money until five years from now. You decide you will put the money into a bank account compounding monthly for that period of time. If your goal is to have **$10,000** when the investment matures, what APR do you need to achieve your goal?

A =

P =

r =

n =

t =

Student Assessment: Solutions

Saving money and planning for your future are not only smart things to do, they're essential! Making smart decisions about investments begins by educating yourself about what types of investments are available to you and which will work the best for you in your particular situation.

The compound interest formula is a tool you can use to evaluate different investments. Make it work for you.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Goal: You want to have **$10,000** saved in a five-year time frame.

How much principal would you need to put away in a bank account offering a **4%** APR and compounding monthly in order to reach that goal?

A = **10000**

P = **unknown**

r = **4% = 0.04**

n = **12**

t = **5**

$$10000 = P\left(1 + \frac{0.04}{12}\right)^{12x5}$$

$$10000 = P(1.00333)^{60}$$

$$10000 = P(1.220753)$$

$$P = \frac{10000}{1.220753} = \$8,191.67$$

You'd have to invest $8,191.67 to accumulate $10,000 five years from now.

Challenge

You have **$8,250** to invest, and you won't need the money until five years from now. You decide you will put the money into a bank account compounding monthly for that period of time. If your goal is to have **$10,000** when the investment matures, what APR do you need to achieve your goal?

A = **10000**

P = **8250**

r = **unknown**

n = **12**

t = **5**

$$10000 = 8250\left(1+\frac{r}{12}\right)^{12x5}$$

$$\frac{10000}{8250} = \left(1+\frac{r}{12}\right)^{60}$$

$$1.21212121 = \left(1+\frac{r}{12}\right)^{60}$$

$$\sqrt[60]{1.21212121} = \sqrt[60]{\left(1+\frac{r}{12}\right)^{60}} = \left(1+\frac{r}{12}\right)$$

$$1.003211344 = \left(1+\frac{r}{12}\right)$$

$$0.003211344 = \frac{r}{12}$$

0.003211344 x 12 = *r* = 0.038536 = **3.8536%**

You'd need the account to have an APR of 3.8536% to accumulate $10,000 five years from now.